Unsupervised Bayesian Probabilistic Signal Detection for Images in Noisy Environments

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Introduction

Problem:
- Detecting weak signals in image data corrupted with strong, complex noise.

Context:
- Noise characteristics are well understood.
- No knowledge about signal distribution.
- No labels available.

Solution:
Norm. Flow + Bayes' Rule $\Longrightarrow$ novel inference framework

Key Features:
- Explicitly modeling noise properties.
- No assumptions on the signal distribution.
- No labels.

Applications: astronomy, biomedical imaging, climate science

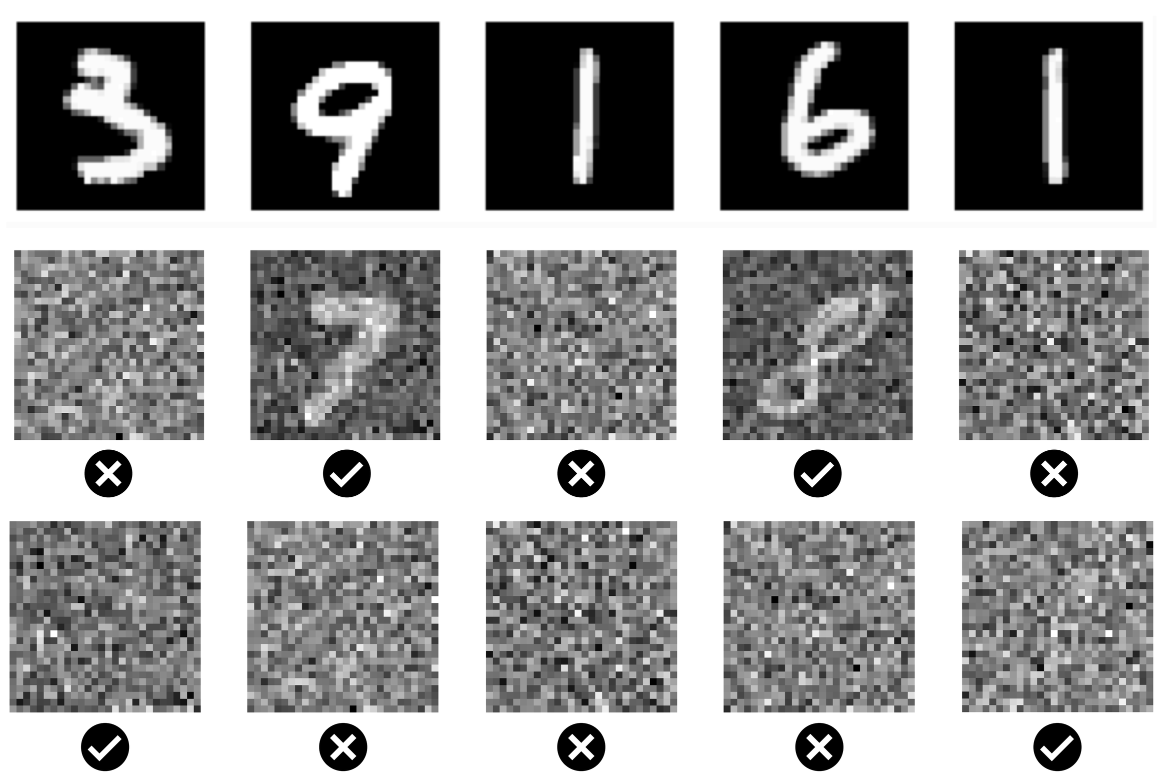

Figure 1: Sample of images with and without signal.

Problem Setup

Model:

$$I = \beta S + N$$

Notation:
- Observed data sample: $I \in \mathbb{R}^d$
- Signal: $S \in \mathbb{R}^d$
- Noise: $N \in \mathbb{R}^d$
- Latent indicator variable: $\beta \in \{0,1\}$

Objective:

$$P(\beta = 1 \mid I)$$

Assumptions

Assumption 1: $P(S,N) = P_S(S)\,P_N(N)$

Assumption 2: $P(\beta = 1) = 1 - P(\beta = 0) = \lambda$

Assumption 3: $P(N \mid \beta) = P_N(N)$

Assumption 4: known $N \overset{iid}{\sim} P_N$

Assumption 5: unknown but fixed $S \overset{iid}{\sim} P_S$

Assumption 6: Labels are not available.

Inference Method

$$P(\beta = 1 \mid I) = 1 - P(\beta = 0 \mid I) = 1 - \frac{P(I \mid \beta = 0)P(\beta = 0)}{P(I)}$$

Challenge:

$$P(I) = P(I \mid \beta = 1)P(\beta = 1) + P(I \mid \beta = 0)P(\beta = 0)$$

Solution: normalizing flows, specifically **Masked Autoregressive Flows (MAF)[2]**, for exact likelihood estimation of I

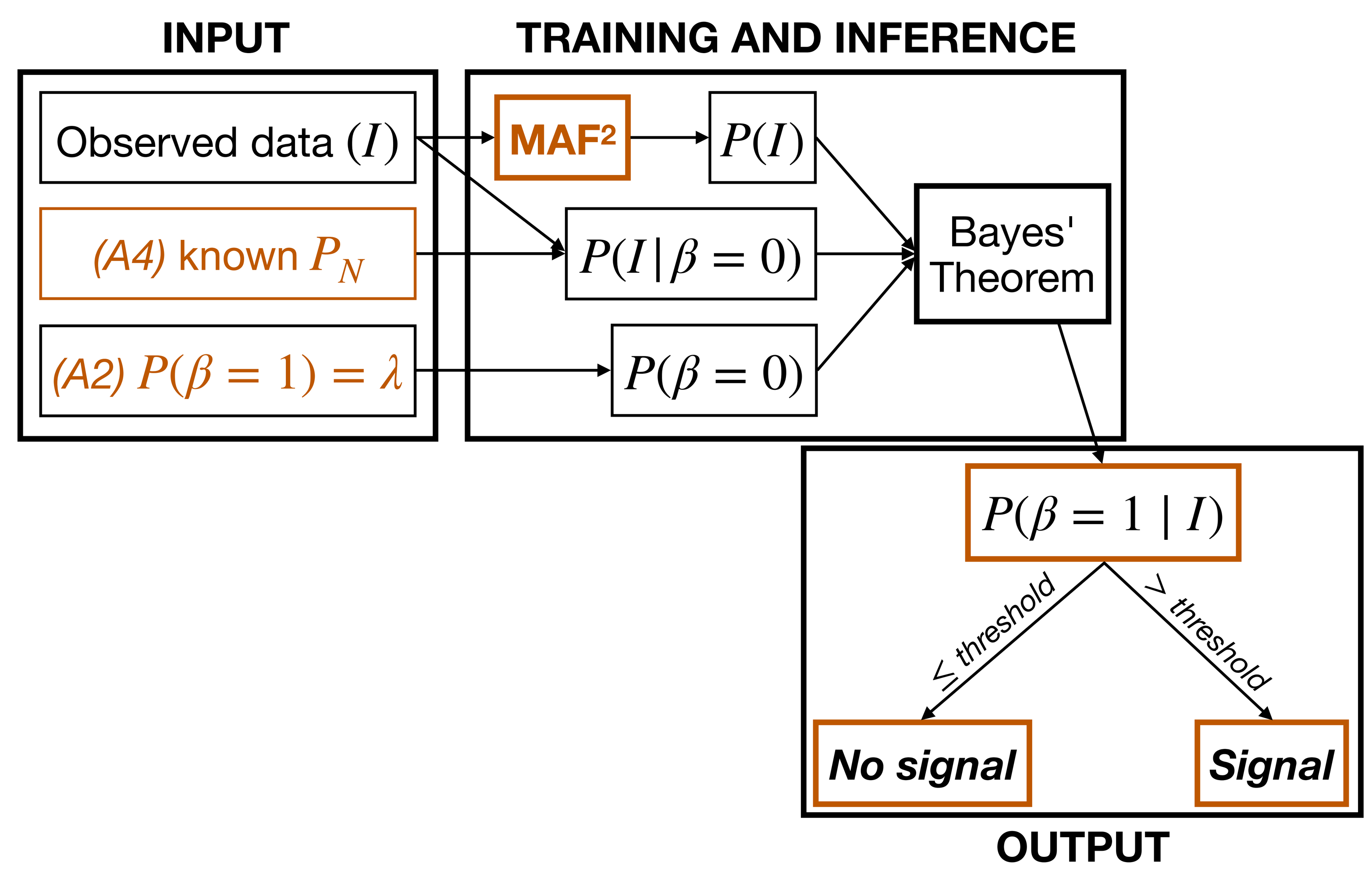

Figure 2: Visualization of the proposed inference framework.

Experiments and Results

Tested SNR	784×10^{-3}	784×10^{-2}	784×10^{-1}	784×10^{0}
Noise level	Very low	Very low	Low	Moderate

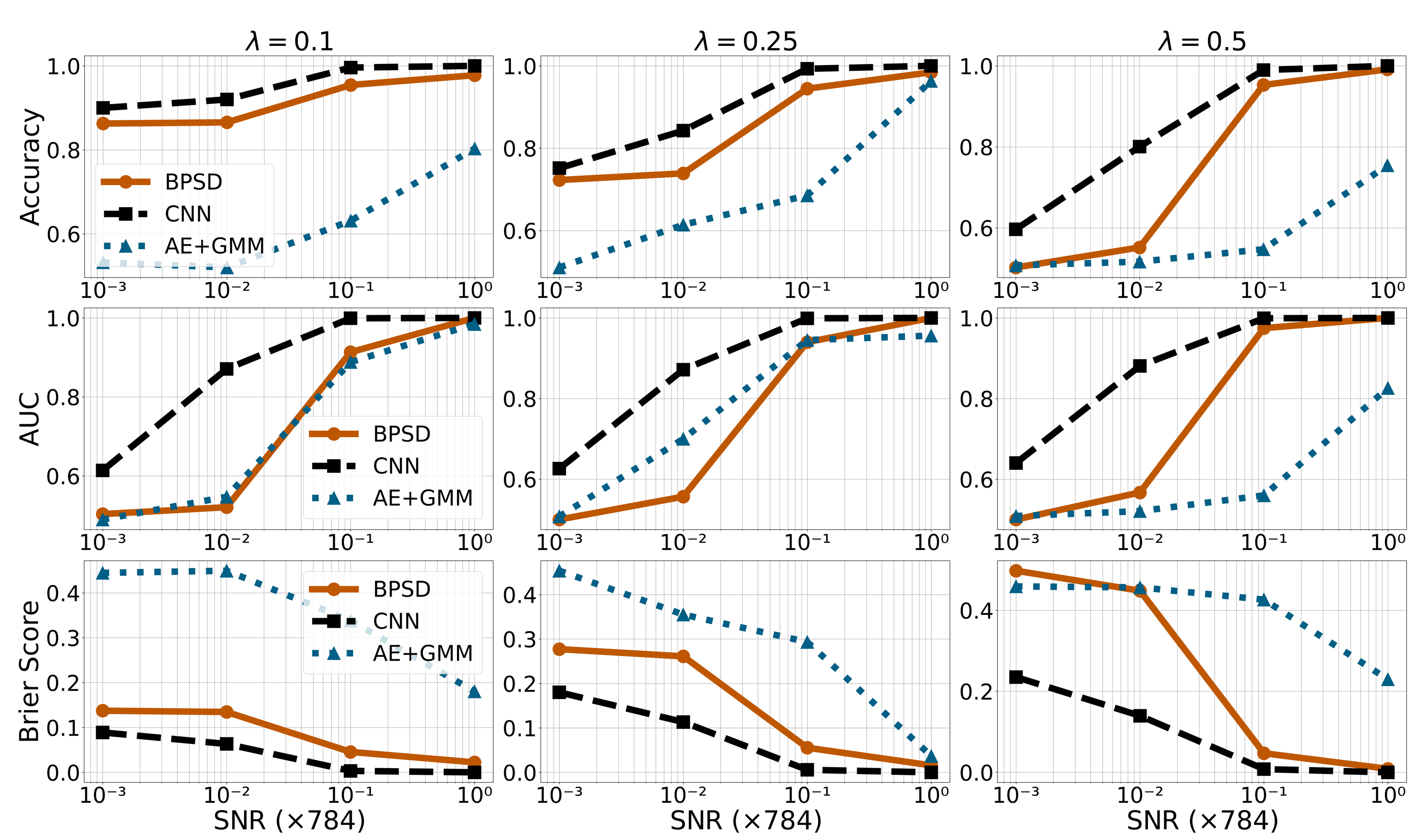

Figure 3: Classification performance when noise is uncorrelated with constant variance, $N \overset{iid}{\sim} \mathcal{N}(0,\sigma_N^2)$.

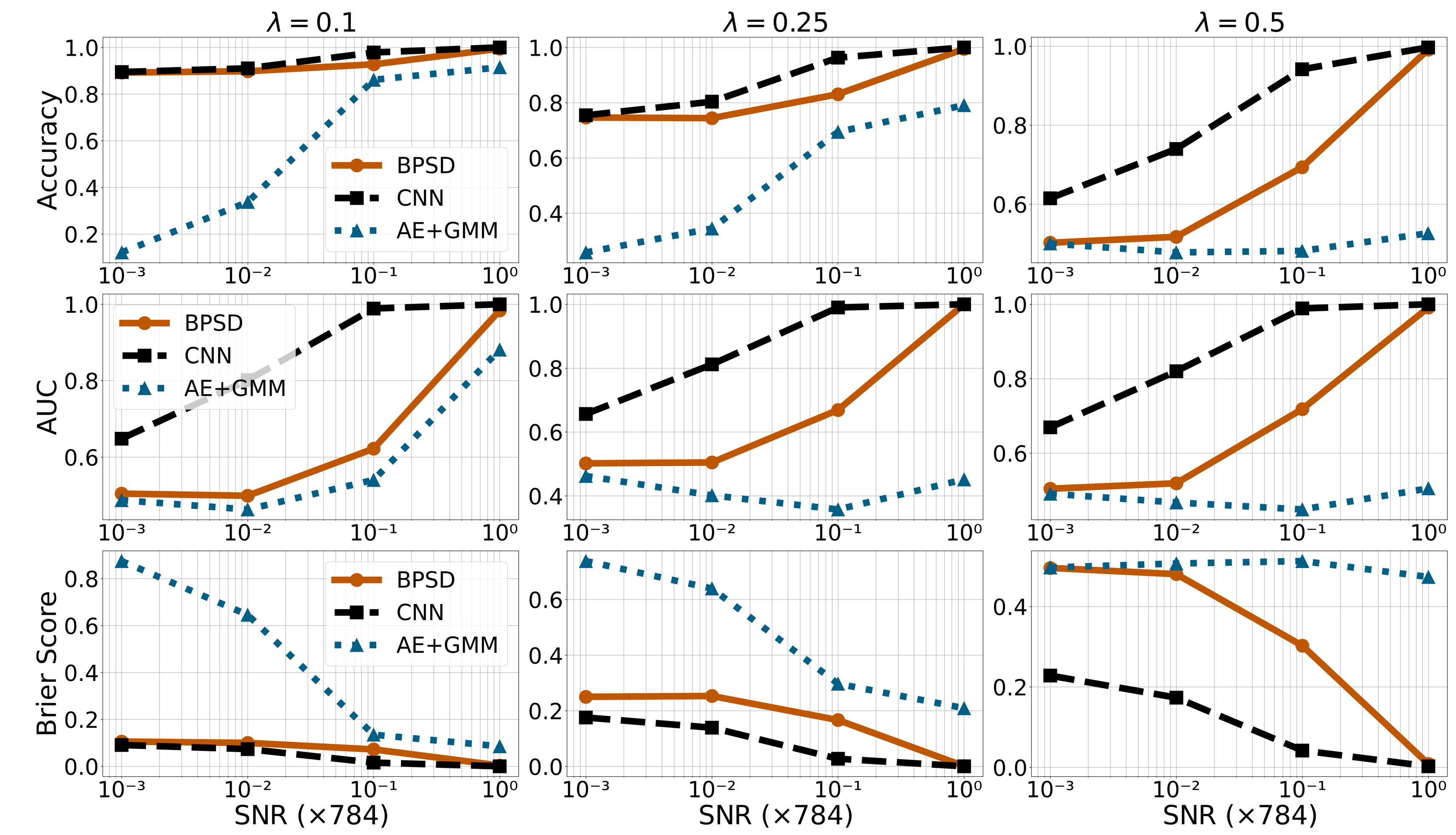

Figure 4: Classification performance when noise is uncorrelated with changing variance, $N_i \overset{iid}{\sim} \mathcal{N}(0,\sigma_{N_i}^2)$.

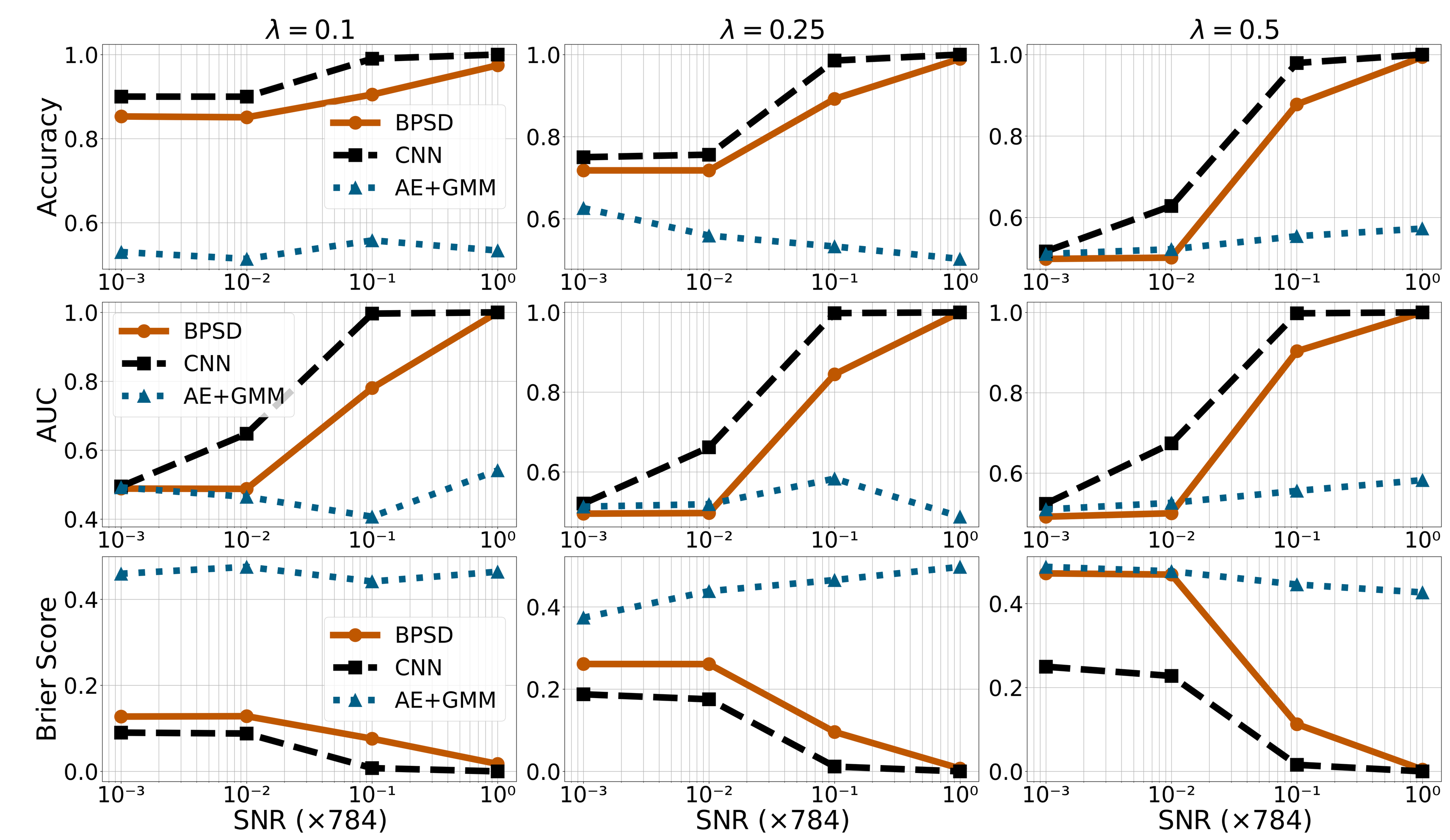

Figure 5: Classification performance when noise is correlated, $N_{1:n} \sim \mathbf{MVN}(0,\boldsymbol{\Sigma}_N^2)$.

Conclusion

Contribution: **inference framework for signal detection in noisy environments**.
How?
- **Norm. Flow (MAF) + Bayes' Theorem**
- No labels.
- No assumptions on signal distribution.
- Leveraging the known noise distribution.

Result:
- On par with traditional supervised approaches.
- Matched or surpassed traditional unsupervised methods.

References

[2]George Papamakarios, Theo Pavlakou, and Iain Murray. Masked autoregressive flow for density estimation. In Advances in Neural Information Processing Systems, pages 2338–2347, 2017.